MISCELLANEOUS.

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173. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

If n is odd, prove the following: $\pm1=[(-1)^{1/n}+(-1)^{-(1/n)}][(-1)^{2/n}+(-1)^{-(2/n)}][(-1)^{3/n}+(-1)^{-(3/n)}]\dots[(-1)^{(n-1)/2n}+(-1)^{-(n-1)/2n}]$
$\pm\sqrt{n}(-1)^{(n-1)/4}=[(-1)^{1/n}-(-1)^{-(1/n)}][-(-1)^{2/n}-(-1)^{-(2/n)}][(-1)^{3/n}-(-1)^{-(3/n)}]\dots[(-1)^{(n-1)/2n}-(-1)^{-(n-1)/2n}]$.

Solution by the PROPOSER.

Delete the factor $(-1)^{(n-1)/4}$ from the first member of the second expression. To resolve into factors the expression $x^n-1=0$. When n is odd,

$$x^n=1=\cos 2m\pi\pm i\sin 2m\pi=(-1)^{\pm 2m},$$

since $\cos 2m\pi=\frac{1}{2}[(-1)^{2m}+(-1)^{-2m}]$, $\sin 2m\pi=\dfrac{1}{2i}[(-1)^{2m}-(-1)^{-2m}]$.

Giving m the values 0, 1, 2, 3, etc.,

$$x=(-1)^{\pm 0},\ (-1)^{\pm(2/n)},\ (-1)^{\pm(4/n)},\ \dots,\ (-1)^{\pm(n-1)/n}.$$

$\therefore$ The roots are 1, $(-1)^{\pm(2/n)}$, $(-1)^{\pm(4/n)}$, ..., $(-1)^{\pm(n-1)/n}$, and the factors are $(x-1)$, $[x-(-1)^{2/n}][x-(-1)^{-(2/n)}]$, $[x-(-1)^{4/n}][x-(-1)^{-(4/n)}]$, ..., $[x-(-1)^{(n-1)/n}][x-(-1)^{-(n-1)/n}]$.

$\therefore x^n-1=(x-1)[x^2-x(-1)^{2/n}-x(-1)^{-(2/n)}+1][x^2-x(-1)^{4/n}$
$-x(-1)^{-(4/n)}+1]$, ..., $[x^2-x(-1)^{(n-1)/n}-x(-1)^{-(n-1)/n}+1]$.

If $x=-1$, $(x^n-1)/(x-1)=1$.

$\therefore 1=[2+(-1)^{2/n}+(-1)^{-(2/n)}][2+(-1)^{4/n}+(-1)^{-(4/n)}]$,
..., $[2+(-1)^{(n-1)/n}+(-1)^{-(n-1)/n}]$,
$=[(-1)^{1/n}+(-1)^{-(1/n)}]^2[(-1)^{2/n}+(-1)^{-(2/n)}]^2$,
..., $[(-1)^{(n-1)/2n}+(-1)^{-(n-1)/2n}]^2$.

$\therefore \pm1=[(-1)^{1/n}+(-1)^{-(1\div n)}][(-1)^{2\div n}+(-1)^{-(2\div n)}][(-))^{3\div n}$
$+(-1)^{-(3\div n)}]\dots[(-1)^{(n-1)\div 2n}+(-1)^{-(n-1)\div 2n}]$.

If $x=1$, $(x^n-1)/(x-1)=x^{n-1}+x^{n-2}+\dots+x+1=n$.

$\therefore n=[(-1)^{1\div n}-(-1)^{-(1\div n)}]^2[(-1)^{2\div n}-(-1)^{-(2\div n)}]^2$,
..., $[(-1)^{(n-1)\div 2n}-(-1)^{-(n-1)\div 2n}]^2$.

$\therefore \pm\sqrt{n}=[(-1)^{1\div n}-(-1)^{-(1\div n)}][(-1)^{2\div n}-(-1)^{-(2\div n)}][(-1)^{3\div n}$
$-(-1)^{-(3\div n)}]\dots[(-1)^{(n-1)\div 2n}-(-1)^{-(n-1)\div 2n}]$.

175. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

If x and z are connected by the relation $z=zf(x)+x\phi(z)$, find the value of $f(z)$ in the form of a power series in x with constant coefficients. In particular, give such a value of z when $z=z\sin x+x\cos z$.